

The restriction principle and commuting families of Toeplitz operators on the unit ball

Matthew Glenn Dawson (CIMAT)

Abstract

On the unit ball B^n we consider the weighted Bergman spaces H_λ and their Toeplitz operators with bounded symbols. It is known from our previous work that if a closed subgroup H of $\widetilde{\text{SU}(n, 1)}$ has a multiplicity-free restriction for the holomorphic discrete series of $\widetilde{\text{SU}(n, 1)}$, then the family of Toeplitz operators with H -invariant symbols pairwise commute. In this work we consider the case of maximal abelian subgroups of $\widetilde{\text{SU}(n, 1)}$ and provide a detailed proof of the pairwise commutativity of the corresponding Toeplitz operators. To achieve this we explicitly develop the restriction principle for each (conjugacy class of) maximal abelian subgroup and obtain the corresponding Segal-Bargmann transform. In particular, we obtain a multiplicity one result for the restriction of the holomorphic discrete series to all maximal abelian subgroups. We also observe that the Segal-Bargmann transform is (up to a unitary transformation) a convolution operator against a function that we write down explicitly for each case. This can be used to obtain the explicit simultaneous diagonalization of Toeplitz operators whose symbols are invariant by one of these maximal abelian subgroups. This is joint work with G. Olafsson and R. Quiroga-Barranco.